Reprinted by permission from IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS
Vol. SMC-2, No. 2, April 1972, pp. 139-144

A Critique of Forrester's Model of an Urban Area

JAMES N. GRAY, DAVID PESSEL, STUDENT MEMBER, IEEE, AND PRAVIN P. VARAIYA, MEMBER, IEEE

***Abstract*—A group of new experiments which we have performed using Forrester's model are presented: a study of rent control, effects of changing key parameters, and a study of changing labor requirements which explores the implications of exogenous variables for the model. A second section presents specific criticisms of Forrester's model, an economic evaluation of the model, and ends with some general remarks concerning validation and extension of large models. In order to make this paper accessible to a wide audience, the paper begins with a tutorial description of Forrester's work.**

I. Description of the Model

THE MAJOR premise of the work of Forrester is that it is often possible to understand and to express the properties of the components of a large social system in mathematical terms and by so doing to describe and predict the total system behavior. He has presented his ideas in the books, *Industrial Dynamics* [2] and *Urban Dynamics* [3].

Most earlier work on modeling cities had centered on either a very specific study of some aspect of an urban area (e.g., land use or power distribution) or presented an input–output matrix for a city. Input–output analysis is a zeroth order approximation to a city. It allows for only linear coupling among state variables. Forrester uses highly nonlinear difference equations which allow much more complex interrelations between elements of the system.

Manuscript received October 5, 1971. This work was supported by the NSF under Grant GK-10656X. This paper was presented at the Second Annual Pittsburgh Conference on Modeling and Simulation, Pittsburgh, Pa., March 29–30, 1971.
J. N. Gray was with the Department of Computer Science, University of California, Berkeley, Calif. 94720. He is now with the General Sciences Department, IBM Thomas J. Watson Research Center, Yorktown Heights, N.Y. 10598.
D. Pessel and P. P. Varaiya are with the Department of Electrical Engineering and Computer Sciences and the Electronics Research Laboratory, University of California, Berkeley, Calif. 94720.

Forrester's main interest is in modeling the city in the abstract. He is not interested in fitting his model to any particular city or set of data but rather is interested in discovering the essential aspects (state variables) of the city and expressing the relationships between these states in mathematical terms.

A. Overview of Model

Forrester's city has three sectors: business, housing, people. Each sector is further decomposed into three categories (roughly, good, OK, bad) so that one obtains the following states:

Business:	New	Mature	Declining
Housing:	Premium	Worker	Low cost
People:	Managers	Workers	Underemployed.

Forrester assumes that the city is confined to a fixed land area and that businesses and houses compete for this land. In doing this Forrester uses the business building as a proxy for the business. He is thus able to model the business by the age of its plant. His model of the business sector is that new business plants are either spun off from existing plants or are attracted to the city from outside. These plants age and house mature businesses and eventually age and house declining businesses. Ultimately the buildings are demolished. Each class of business has different employment needs. New businesses require more managers than declining businesses, for example.

Housing has a similar structure: premium housing is built and eventually declines to worker housing, worker housing is also built and then declines to underemployed housing after some period of time. This is the main source of underemployed housing. Underemployed housing degrades slowly and eventually is demolished.

The decision of a new business to come to the city, of a builder to build a house, or of a person to come to the city or to leave is determined by the attractiveness of the city relative to its environment. The model supposes that there are an infinite number of people of each category, houses of each category, and of businesses sitting outside the city continually asking themselves if they would be better off moving into the city. Their decision is based on their perception of the state of the city in comparison to the ambient condition, its taxes, the availability of land, jobs, labor, and housing. Each category of each sector bases its decision on different criteria. For example, business and managers want low taxes, and underemployed people want high taxes.

B. Detailed Description of the Tax Structure

Forrester gives a detailed description of the dynamics governing the underemployed population [3, pp. 20–31] We present a brief description of the tax structure here to give the flavor of the model.

The taxes collected (TC) are the product of the assessed value (AV) of the city multiplied by the normal tax assessment (TAN) (the ambient tax in the limitless environment) multiplied by the tax ratio (TR) of the city (with respect to the ambient tax): $TC = AV \cdot TAN \cdot TR$. The assessed value is computed by assessing premium houses at \$30 000 per unit, worker housing at \$15 000, low-cost housing at \$5000, and business at \$500 000 for a new business unit, \$300 000 for a mature business, and \$100 000 for a declining business. The normal tax assessment is \$50/year/\$1000.

The tax ratio of the city is a critical variable of the system since it figures prominently in the attractiveness of the city. The way in which it is computed exemplifies the structure of the model. The tax ratio depends on the perceived needed tax ratio. The needed tax ratio is the amount of tax needed to provide services to each sector of the population. In general it is greater than the perceived needed tax ratio for two reasons. First, the perceived needed tax ratio (TRNP) lags behind the needed tax ratio (TRN) by 30 years (exponential time delay). This models an information gap:

$$TRNP_{t+1} = TRNP_t + \frac{(TRN_{t+1} - TRNP_t)}{30}.$$

Also, for political or practical reasons it may not be possible to assess the full tax needed. Forrester models this by having the tax ratio lag the perceived needed tax ratio logarithmically, i.e., $TR \simeq 0.7 \log_2 (TRNP) + 1$. The tax ratio needed is computed by summing the demands of the individuals in each sector of the city. Managers require \$150/person, workers \$200/person, and underemployed \$300/person. Further, managers have five people in their family, workers six people in their family, and underemployed have eight people in their family.

Under normal conditions this means that a professional family contributes \$750/year to the city, while a worker family costs \$450/year, and an underemployed family costs the city \$2150/year (services minus property tax). Each new business plus its employees costs the city \$2500/year, each mature business costs \$6375/year, and each declining industry costs \$9500/year (tax minus demands of employees). Clearly, if the city is to balance its budget, it must provide services below normal or raise its tax-rate. At equilibrium the tax rate is more than double the ambient tax.

That is the tax structure. On the basis of this and 19 other states, people come and go, houses are built, and business arrives or fails to arrive.

C. Forrester's Experiments

Given the model set loose with almost no people, no houses, and no industry, it arrives at an equilibrium state after a few hundred years. This equilibrium state is called a "stagnant condition" by Forrester. His objection to it is that there is large unemployment and a surplus of old structures. He first reviews several popular programs: job program for unemployed, job training for unemployed, block grant to the city, and low-cost housing construction. Each of these makes the city more attractive to the underemployed and so results in an even larger surplus of underemployed people. This in turn causes taxes to go up and business to flee.

He also examines some less popular programs: attract new industry, demolish declining business, demolish slum housing, and discourage housing construction. Each of these causes "favorable" shifts in the equilibrium.

D. General Remarks on Large Systems

Forrester argues persuasively that cities are very complex and that they are counterintuitive. He argues that computers can handle models at least as complex as the models in the minds of city planners and mayors, and that these models being mechanistic allow (require) rigor not commonly found among decision makers. He does not view the model he presents as such a model. He is rightly modest about its usefulness. He does argue, however, that if there is any hope for rational planning, modeling should be pursued as an important tool.

Perhaps the most impressive example of this is a recent exchange between Forrester and Kain [4], [5]. Kain reviewed Forrester's work and, although quite impressed by the general idea, made specific criticisms of Forrester's model (specifically the tax structure). Kain suggested some changes which he believed would produce different results. (One cannot criticize Forrester's conclusions since they follow directly from his hypotheses. One can only criticize his hypotheses.) Forrester followed Kain's suggestions and reported no "significant" changes in the performance of the model.

II. Some New Experiments

In order to experiment with Forrester's model we first implemented a Dynamo to Fortran translator (in Snobol) and then translated Forrester's model to Fortran. Thereafter, we worked entirely within Fortran, its advantages being speed, portability to other institutions, and the availability of a graphical I/O package. The latter proved to be an important asset. A typical experiment compiles in 10 s, runs in 5 s, and requires 20 s to produce plots. A run costs less than $5. All our work was done on a CDC 6400 comparable to an IBM 360/65. We were not hindered by the fact that Fortran is a poor simulation language since Dynamo is even worse.

It is considerably easier to invent experiments than to interpret their results. For this reason our experiments were carefully planned. In each case we had some hypothesis about what should happen. When the hypothesis was verified by the model, this added credibility to the model and to the hypothesis. If the model produced surprising results, the experiment and the results were each carefully examined to determine which assumptions were violated.

A. Rent Control

Our first experiments added to the credibility of Forrester's model. We investigated the institution of rent control in New York City. Rent control was imposed in New York during World War II on all existing housing units (it does not apply to units built after the law was passed). To state the law simply, it allows the landlord to raise the rent on a unit in only two cases: a) when a tenant moves the rent may be raised up to 15 percent; and b) when improvements are made the rent may be temporarily raised to amortize the cost of the improvements [6], [8].

Rent control has greatly retarded the rise in rents in older buildings in New York. It has thus made the units accessible to people who might not ordinarily be able to afford them. The expansion of Harlem (and other ghetto areas) has been stimulated by the consequent availability of low-cost housing. Harlem has grown from an area bounded by 125th and 150th Sts. in 1945 to its present boundaries of 96th and 170th Sts. The boundaries are very fuzzy—some people might place them as far north as 181st St. and as far south as 72nd St.; however, no one disagrees that there has been a dramatic increase in the size of Harlem. Almost all the housing in this area is old and thus rent controlled.

The dynamics of what happened in Harlem seem to be in part that as housing became accessible to the underemployed blacks due to lower rents they quickly absorbed it. As more and more underemployed moved into the area it quickly became an underemployed area. (Forrester's model does not include this spatial dynamic.) The labor segment was, of course, often paying a rent below its means. Thus the workers moved differentially out of Harlem (typically to the suburbs). In this way one of the large faults of a city was continually worsened. Labor housing became underemployed housing, the underemployed were attracted in even larger numbers by the availability of rent controlled housing, and their population increased. Workers left the area and their population—by which a city is often judged—decreased. Of course rent control added some premium housing to the worker housing market, but not in sufficient quantity to offset the drain into underemployed housing because there was so much less premium housing than worker housing.

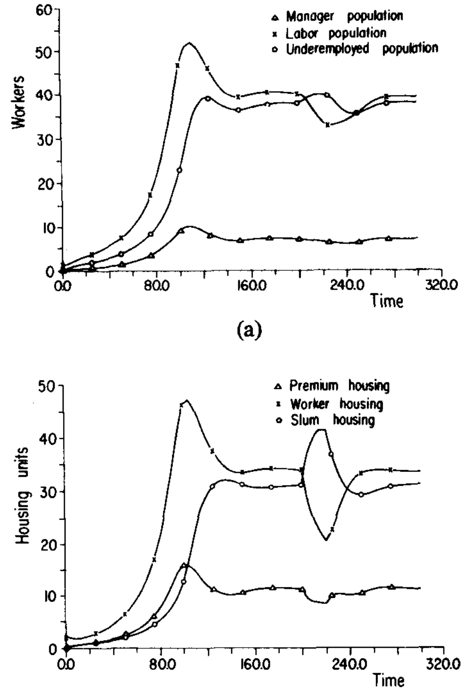

Fig. 1. (a) Population rent control. (b) Housing sector rent control

This was our analysis of the impact of rent control on the evolution of Harlem. Since the Borough Manhattan (which includes Harlem) is a classic example of a core city, this gave us an excellent opportunity to test Forrester's model. Would it behave as Manhattan had?

We assumed that each year for 20 years 5 percent of the houses in each class "filtered" down to the next lower class due to rent control (premium housing (PH), worker housing (WH), and underemployed housing (UH)): $PH = 0.95 * PH$, $WH = 0.05 * PH + 0.95 * WH$, $UH = 0.05 * WH + UH$. This was done at Time = 200 years (i.e., at equilibrium) for 20 years. Fig. 1 shows the population and housing curves. At first the city grows, but gradually all rent control housing is demolished, and the system returns to equilibrium. Currently, after 25 years of rapid growth, the area known as Harlem is declining in population, although it is still growing at its periphery. This corresponds to the predictions of the model.

We do not mean to suggest that rent control is the reason that Harlem grew as it did. Many other urban ghettos have behaved similarly without any rent control. The experiment and our analysis simply attempt to explain what impact rent control has had where it applies. In light of the work of Pack [9] and Babcock [10] the importance of housing to the attractiveness of the city is minimal. We believe that the model modified to their specifications would behave similarly although not as spectacularly.

Further experiments indicated that a possible amelioration of the situation might be to impose a rent control on housing that has reached a certain rent level (measured in terms of cost per room, for example). This level might be the worker housing level. Units charging rent below this level would be allowed to fluctuate. This would reduce the filtration of worker housing into underemployed housing by failing to bring any new housing within reach of the underemployed and would retain labor in the city.

B. Parameter Changes

Several parameter changes were made to the model. The family sizes were changed so that the average family size was 5.2 rather than 6.6. This made no significant change in the response of the model, although there was a relative decline of underemployed people, underemployed housing, and declining industry.

Changes in perception times were attempted. It was conjectured that the long perception times were damping the system and that it might oscillate if they did not exist. Setting all perception times to one year disproved this. The model simply responded more quickly to changes. It appears that perception lags add complexity to the model without improving its predictions.

Many other parameter changes were inadvertantly tried during the debugging of the Dynamo compiler and the keypunching of the model. It was often found that the tables of interpolated values were accessed outside of their defined range. These variations made little difference (typically equilibrium states agreed to eight decimal places).

Only one parameter change shed very much light on the structure of the model. Both the housing and business sector are modeled by the process of filtration. In the business sector new industries become mature businesses which ultimately become declining industries. Although it is generally agreed that filtration (and construction) are the essential dynamics of the housing sector, this decomposition of business is orthogonal to the classical decomposition into administration, service, retail, wholesale, and manufacturing. One suspects that the former decomposition was chosen because of its simplicity rather than for technical reasons. It could be very difficult to model the interaction of the classical sectors, whereas the filtration model can be expressed in three rather simple equations.

We observed that at equilibrium the ratio between the number of units of industry in each sector was roughly the same as the ratio between the mean lifetime of units in each sector. In experimenting we found this to be true in general. Since Forrester laments the excess of declining industry over mature business, it seems appropriate to comment that it stems from his assumption that new enterprises, mature businesses, and declining industries have mean lifetimes of 12, 20, and 33 years, respectively. Frankly, we reject this model of the business sector.

We conclude that the equilibrium of the system stems from the fact that it has a fixed land area thus constraining each of the variables from divergence, and from the fact that the "filtration" model of business and housing strongly constrains their populations by fixing their ratios.

C. An Exogenous Dynamic

We were quite mystified by the fact that Forrester's model of a city is so stable. It converges to an essentially unique equilibrium state very quickly, independent of initial conditions (degeneracy is possible). This is in contrast to another model we have seen which has three (rarely achieved) equilibrium states. It is a model of rural Paraguayan communities due to Otto Smith and his students [1], [7].

Equilibrium plays an essential role in Forrester's theory of urban growth and stagnation. It means that unless the dynamics of the city change, stagnation will continue. He sees the city resisting any changes to its equilibrium state. Even the rather major changes suggested by Kain had no "significant" effect.

The previous section gave an operational explanation of the causes for monostability of Forrester's model. We observe that the central city of the United States in mid-century does not appear to be in equilibrium. In the last 30 years there have been gross population shifts of the middle class to the suburbs; in the last 50 years heavy industry has moved from the central city to be replaced by office buildings. Attempts to model this quickly point out some of the inadequacies of Forrester's model. First, there is no distinction between an office building and a factory in his model. Second, all workers live in the city. Third, there is no precedent in Forrester's model for exogenous variables. We have been unable to understand Forrester's arguments about not needing to consider exogenous variables [3, pp. 17–18].

Despite these difficulties we attempted to model the shift in the labor market within the context of Forrester's model. We argue that the pool of jobs can be dichotomized as skilled–unskilled; that only working class people qualify for skilled jobs, but that both workers and underemployed people compete for unskilled jobs. Further, there is a bias toward hiring workers for unskilled jobs if possible. In our experiment the pool of jobs is computed as JOBS = LDC + LDI, where LDC is labor desired by construction and LDI is labor desired by industry. This is as in Forrester's model. However, we depart by computing the number of unskilled jobs: UNSKILL = NOSKILL(TIME) * JOBS. NOSKILL is a function of TIME giving the fraction of jobs which require no skill. We (somewhat arbitrarily) chose NOSKILL to be a linear interpolation of the function: NOSKILL (0) = 0.90, NOSKILL (100) = 0.80, NOSKILL (200) = 0.40, NOSKILL (300) = 0.30. In some sense this models an industrial and white collar revolution beginning in the city at TIME = 100 and stabilizing at TIME = 200.

We further wanted to model the bias against hiring underemployed people. We assume that only laborers can assume skilled jobs, but laborers and underemployed people compete for unskilled jobs. The ratio LSR = L/(JOBS − UNSKILL) gives some index of the intensity of this competition. If LSR < 1, there is no competition. As LSR rises above 1 the competition increases. We model this by defining the function BIAS of LSR as: BIAS (0) = 1.00, BIAS (1) = 1.00, BIAS (2) = 0.50, BIAS (3) = 0.25. Then the number of underemployed jobs is UJ = BIAS (LSR) * UNSKILL. Remaining jobs go to laborers, so, LJ = JOBS − min (UJ, U).

Examining Fig. 2 shows the results of these changes. The model is no longer stable at 150 years. There are still major shifts taking place, the population and job sectors being the most active. Only after the NOSKILL function stabilizes

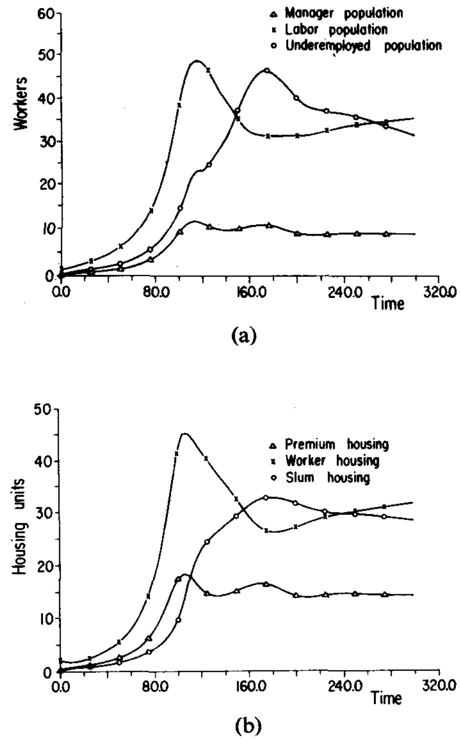

Fig. 2. (a) Population time varying number of skilled jobs. (b) Housing sector time varying number of skilled jobs.

does the model settle down. The introduction of other exogenous variables would presumably have similar effects.

The concept of the equilibrium or "stagnant" city has disappeared entirely. Clearly many other things have changed in the last 250 years. The tax structure of the Massachusetts Bay Colony differs from that of Boston. The dynamics of the city are not static—they themselves vary. Forrester's model does not reflect this. This in part explains its perverse stability.

In light of this one must discount the evolutionary aspects of Forrester's model (evolving a city from a virgin state) and view the (static) dynamics of the model as the dynamics of a central city in the United States in mid-century.

The loss of the concept of equilibrium is nontrivial. With equilibrium it makes no difference when (in time) an experimental program is started. The city is always in the same state and has the same dynamics, so it will respond in the same way. Without equilibrium this is not the case. For example, the effects of a job program initiated during a depressed period may be substantially different from the effects of the same program initiated during periods of economic boom.

III. Conclusions

The most common criticism of Forrester's work is that it does not model the suburbs. This is only one aspect of the shortcomings of Forrester's assumption of a limitless environment. Since 70 percent of U.S. population lives in an urban area, it would be better to view "the city" as the aggregation of all cities, rather than just one with no people on the outside. Otherwise one finds himself trying to get rid of underemployed people by making the city unattractive for them. Forrester does not consider the question of where they go when they leave the city (answer: New York, Chicago, Los Angeles). In this respect his conclusions may be good advice for a mayor but bad advice for the Secretary of the U.S. Department of Health, Education, and Welfare. His model encourages local (citywide) optimization rather than global (national) optimization.

This naturally leads to the next criticism, which is that although Forrester is very precise about how many people are in the average family (6.6), he never is precise about an objective or cost function. He talks vaguely about a stagnant city but never defines a healthy city. The one variable which seems to interest him most is the mobility of underemployed into the labor class (UM). This reflects his view of the city as a melting pot and social converter. We suggest that a sophisticated and definite statement of a healthy city is needed. This definition should form an integral part of the model and the process of judging the virtues of various programs.

A third criticism is that Forrester draws general conclusions from his specific model. His model is a highly damped (perception times ~ 20 years) continuous time-invariant system. The fixed land area constrains it from divergence, the damping constrains it to continuity, and the filtration model of business and housing causes a unique equilibrium (except when a variable goes to zero in which case one gets a degenerate equilibrium). He does not model pump-priming or seed programs. These are small changes in the system which have a large impact on the equilibrium state. He says that [3, p. 75] "A positive program to generate some rate of flow will change the system balance and cause a depression in the normal processes generating that same rate. The new program is only partly effective because much of it displaces normal processes." He also talks a great deal about the insensitivity of his model to parameter changes (except some critical parameters). Again this is a result of his model. One should not conclude from his model that it is true of cities in general. Other data must be presented to verify that. For example, one experiment we have done shows that if the structure is not time invariant (e.g., the job market at TIME = 0 is different from the market 250 years later), then the concept of static equilibrium disappears from his model.

One may criticize Forrester for his ad hoc approach. He does not explicitly present any underlying theory for his model. The implicit theory appears to be too primitive for such an ambitious model. For example, the two sectors, business and construction, have production functions which show constant returns to scale, and use factors, land and labor, in fixed proportions for the entire period of 300 years. No substitution between factors is permitted, and capital requirements are ignored. Mobility of labor seems to be reasonably modeled, although even here the assumption of time invariance of the underlying causal relations is highly suspect. The assumption of fixed land area ignores the economic fact that capital can be substituted for land (a flat Empire State Building would occupy 100 square blocks) and the historical fact that most cities have extended their geographical boundaries. On the other hand, the rate of new activity is made to depend in part on the overall level of activity, so that interesting accelerator-type relations are built into the model. As a final comment one may question the disaggregation of business into new enterprises, mature business, and declining industry, with each firm passing through this life cycle in a mechanistic manner. Although such a classification may be suggestive for some purposes, it is not very easily convertible into an operational definition when one considers the way in which industrial data is collected and presented. In sum, we get the overall impression that the theoretical foundations of the model received minimal attention.

Forrester's disinterest in modeling a particular city aggravates this. There is no attempt to match the initial state, parameters, or behavior of the model to a real city. Thus there is no way to correlate the behavior of his model with the behavior of a real city. A more rational approach is to pick key variables, empirically establish their relationships, and then tune the model to predict the past performance of the system. Finally, to close the loop, the model is used to predict the future. Any divergent results then feed back to correct the model.

This raises the question of making changes in the model. For example, if the business sector is modified, how is the new model validated? Is it necessary to run all of Forrester's experiments again and carefully check their results? Clearly the answer is yes. Now suppose ten man-years are invested in developing a "better" model. Then one day a small change is made. Is it necessary to repeat the ten years of validation procedures?

If there is to be any hope of building on the work of others, models must be designed in a modular fashion. One should be able to say: "This is a model of the housing sector. If you change the business sector, it won't affect the validity of the housing sector model." The development of modules will also allow more complex models. Forrester's model is a gestalt; it was conceived and built by one man. It might be said that it is as far as one man can go. More complex models will have to be decomposed into sectors, with a different group working on each sector.

In order to achieve this it will be necessary to develop rational validation schemes for modules. One cannot hope to test the module explicity for each situation. If no such validation is possible, it brings into question the major premises of Forrester's work: that it is possible to understand and to express the local properties of a large system in mathematical terms, and by so doing to describe its global behavior.

References

[1] D. Daetz, "Energy utilization in a farm community; non-industrial standard of living model," Ph.D. dissertation, Dep. Elec. Eng., Univ. California, Berkeley, 1968.

[2] J. W. Forrester, *Industrial Dynamics*. Cambridge, Mass.: M.I.T. Press, 1961.

[3] ——, *Urban Dynamics*. Cambridge, Mass.: M.I.T. Press, 1969.

[4] ——, "Overlooked reasons for our social troubles," *Fortune*, Dec. 1969.

[5] J. Kain, "A computer version of how a city works," *Fortune*, Nov. 1969.

[6] E. Olsen, "The effects of a simple rent control scheme on a competitive housing market," Rand Corp. Rep. P4257, Dec. 1969.

[7] H. Pande, "Aid effectiveness in non-mechanized agriculture: computer simulation," M.S. research project, Dep. Elec. Eng., Univ. California, Berkeley, 1969.

[8] G. Steinlieb, "New York City housing: a study in immobilism," *Public Interest*, no. 16, Summer 1969.

[9] J. R. Pack, "Models of population movement and urban policy," this issue, pp. 191–195.

[10] D. L. Babcock, "Assumptions in Forrester's *Urban Dynamics* model and their implications," this issue, pp. 144–149.