Exercise 1.

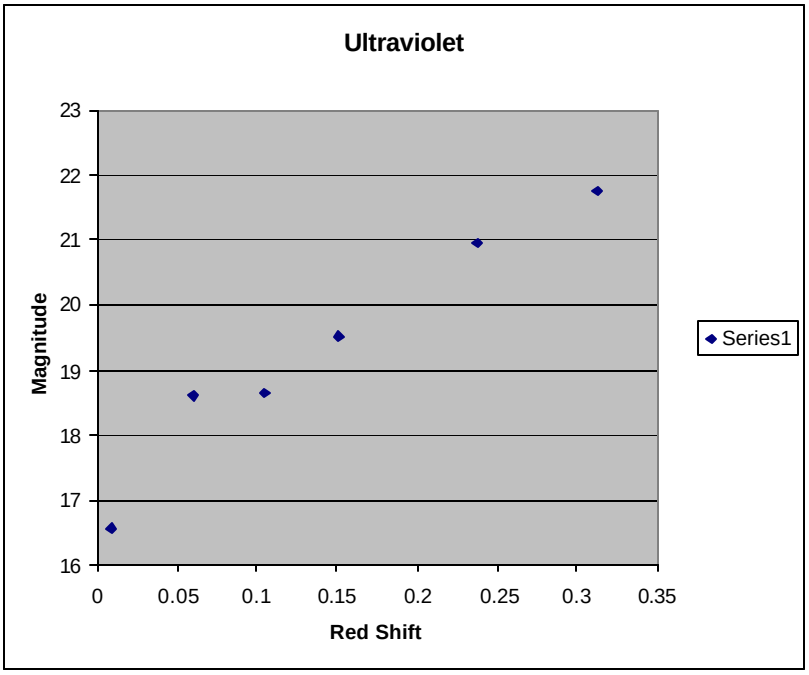
	Magnitude				
Galaxy ID	Ultraviolet	Green	Red	Infrared A	Infrared B
1970729646162088	16.56	15.47	15.14	14.75	14.75
1970862784315442	18.61	16.33	15.36	14.84	14.58
2255030735995063	18.64	16.81	15.88	15.48	15.14
568774202097915	19.52	17.61	16.54	16.08	15.78
850262602481913	20.97	18.70	17.22	16.69	16.37
2255030437282027	21.75	19.82	18.16	17.56	17.14

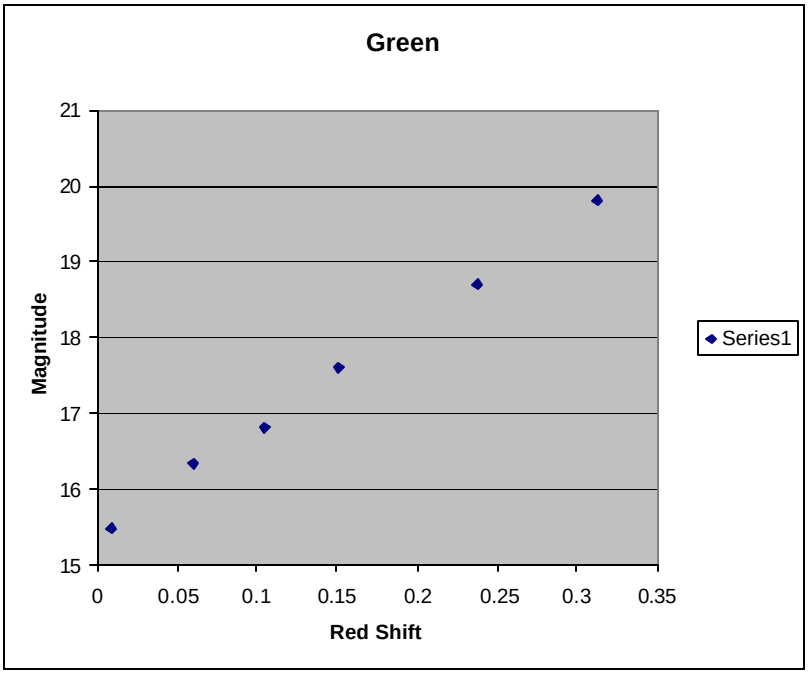
Question 1. As objects get farther away, their light appears dimmer, just as a candle across the room looks dimmer than a candle held at arm's length. If we assume that all galaxies output the same amount of light, then any difference in the galaxy's apparent brightness will be due to its distance from Earth. Magnitude measures the apparent brightness of a galaxy. Therefore, different magnitudes correspond to different distances from Earth.

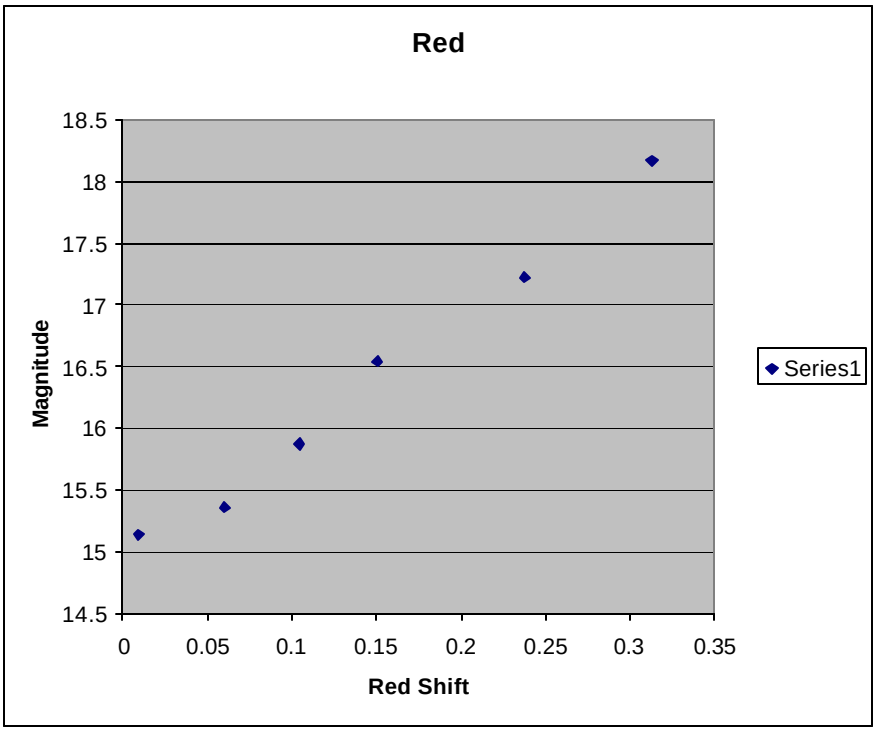
Exercise 2.

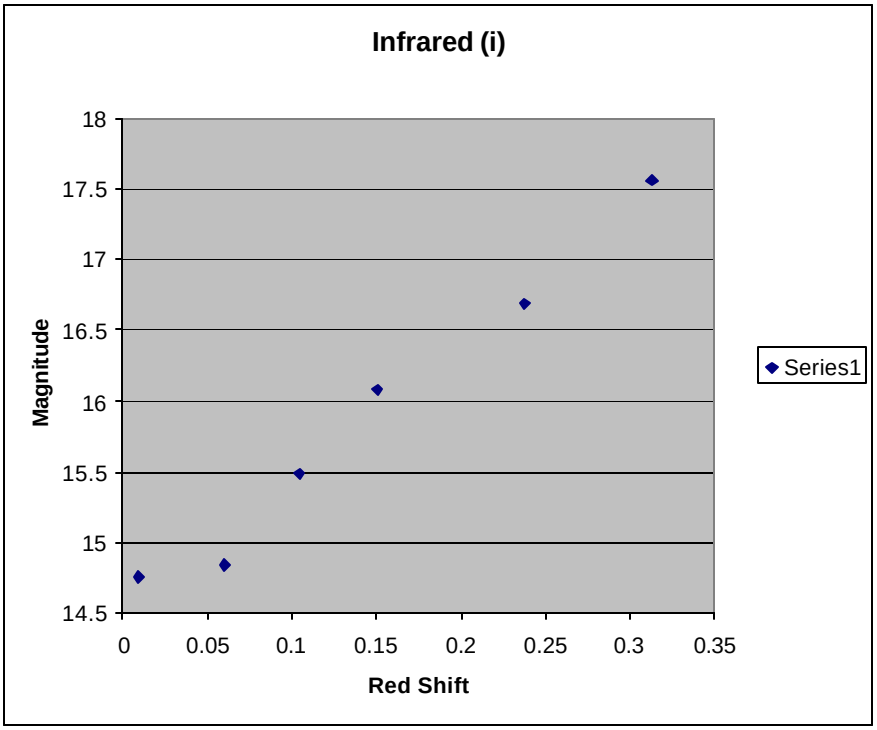
Galaxy ID	Redshift
1970729646162088	0.009
1970862784315442	0.06
2255030735995063	0.105
568774202097915	0.151
850262602481913	0.238
2255030437282027	0.313

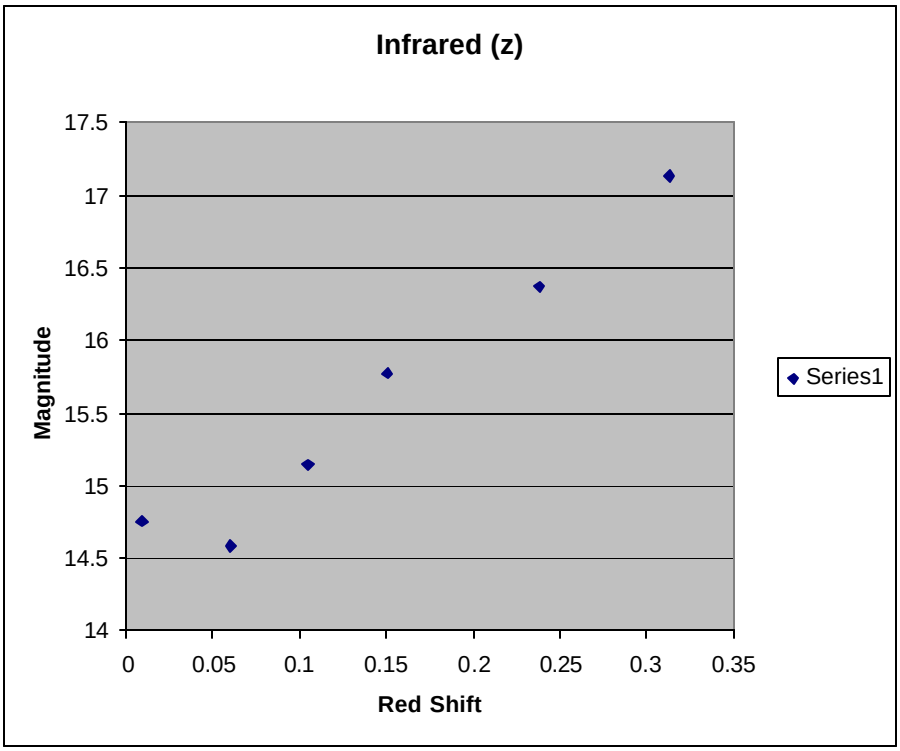
Exercise 3.


Ultraviolet
Magnitude
23
22
21
20
19
18
17
16
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
Red Shift
Series1


Green
Magnitude
21
20
19
18
17
16
15
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
Red Shift
Series1


Red
18.5
18
17.5
17
16.5
16
15.5
15
14.5
Magnitude
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
Red Shift
Series1


Infrared (i)
18
17.5
17
16.5
16
15.5
15
14.5
Magnitude
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
Red Shift
Series1



Exercise 4.

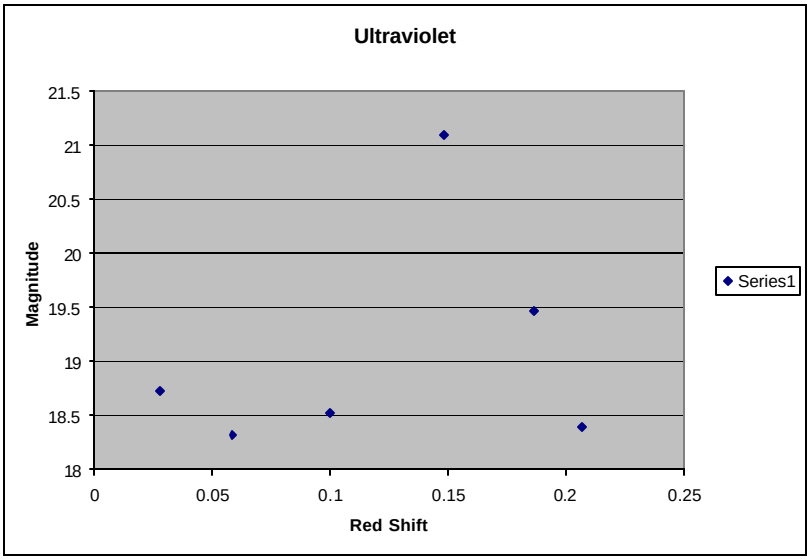
Wavelength	**Fit**
Ultraviolet	95.0%
Green	99.9%
Red	99.0%
Infrared A	98.1%
Infrared B	95.5%

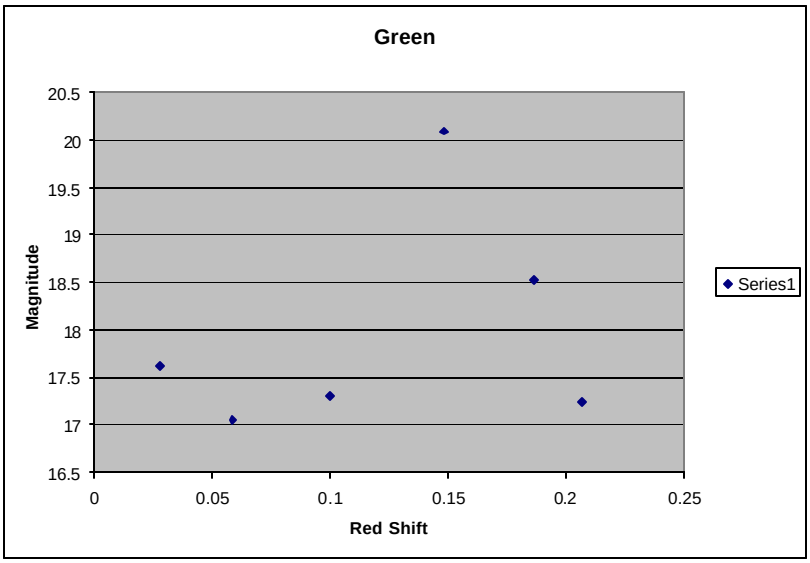
No matter which wavelength you chose to look at, a straight line is a good fit to your data.

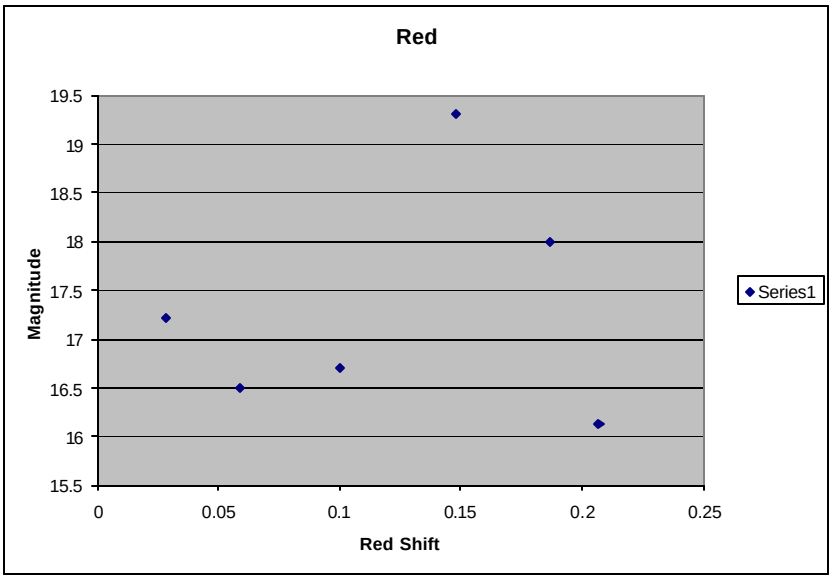
Exercise 5.

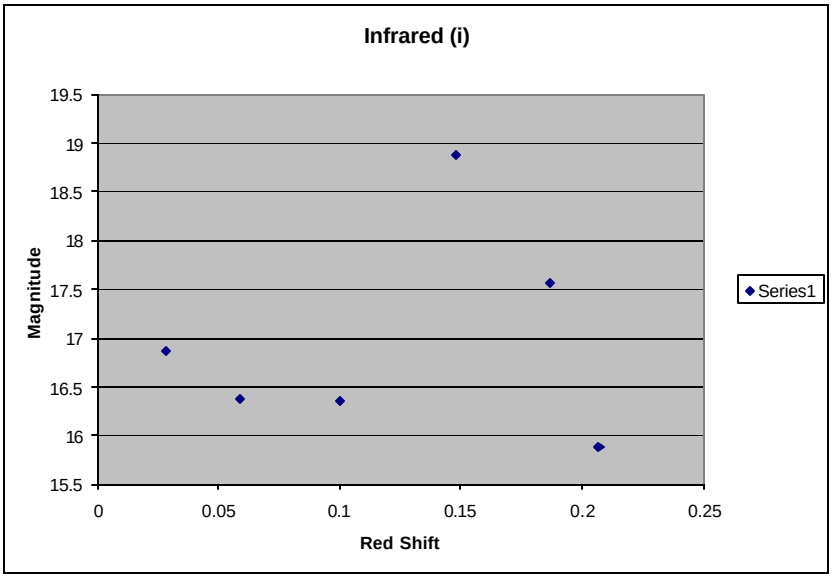
Galaxy ID	Ultraviolet	Green	Red	Infrared A	Infrared B	Redshift
2255047396163863	18.72	17.62	17.22	16.88	16.67	0.028
2255047397081521	18.31	17.04	16.50	16.38	16.18	0.059
1970729917612234	18.52	17.29	16.71	16.35	16.10	0.1
1970862519025754	21.10	20.10	19.32	18.88	18.69	0.148
1970863056224378	19.599	18.634	18.07	17.624	17.487	0.187
1970863324266615	18.39	17.23	16.14	15.87	15.66	0.207

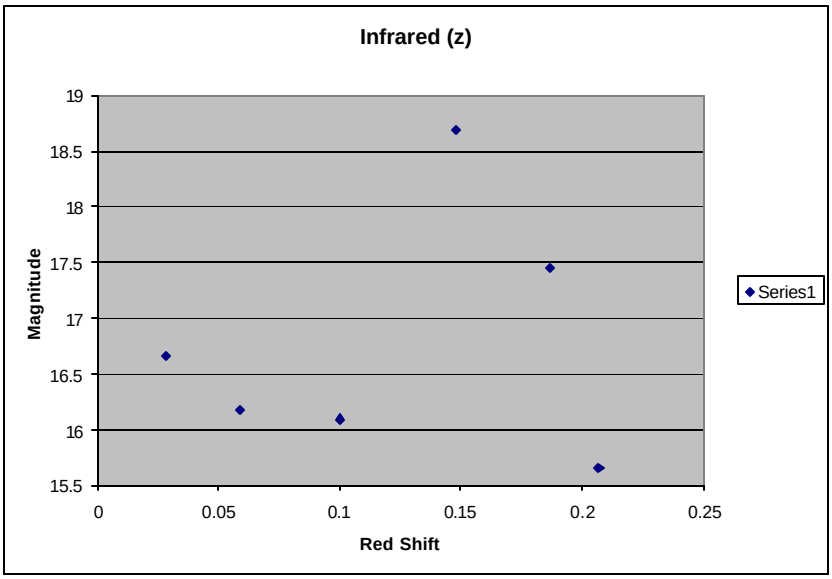
Exercise 6.


Ultraviolet
21.5
21
20.5
20
19.5
19
18.5
18
Magnitude
0
0.05
0.1
0.15
0.2
0.25
Red Shift
Series1


Green
20.5
20
19.5
19
18.5
18
17.5
17
16.5
Magnitude
0
0.05
0.1
0.15
0.2
0.25
Red Shift
Series1


Red
19.5
19
18.5
18
17.5
17
16.5
16
15.5
Magnitude
0
0.05
0.1
0.15
0.2
0.25
Red Shift
Series1


Infrared (i)
19.5
19
18.5
18
17.5
17
16.5
16
15.5
Magnitude
0
0.05
0.1
0.15
0.2
0.25
Red Shift
Series1



Wavelength	Fit
Ultraviolet	8.8%
Green	10.5%
Red	2.8%
Infrared A	1.9%
Infrared B	2.4%

No matter which wavelength you chose to look at, a straight line is not a good fit to your data.

Exercise 7. The answers will depend on which wavelength you chose to examine to get the magnitudes. In any wavelength, you should calculate distances *d* from the magnitudes *m* with the equation

$$d = \frac{1}{\sqrt{2.51^{-m}}},$$

then create ratios where the nearest galaxy has $d = 1$ and farther objects have distances in the same proportions that you calculated:

$$\frac{d_2}{d_1} = \frac{1}{x}.$$

Solve for x, the normalized relative distance, then repeat for the other galaxies.

The answers below are for the green wavelength. Other wavelengths should give similar relative distances.

Galaxy ID	Magnitude (green light)	Relative distance
1970729646162088	15.47	1
1970862784315442	16.33	1.48
2255030735995063	16.81	1.85
568774202097915	17.61	2.67
850262602481913	18.70	4.42
2255030437282027	19.82	7.40

Exercise 8. From magnitude,

$$d = \frac{1}{\sqrt{2.51^{-m}}}.$$

The quantity 2.51^{-m} is the radiant flux, the amount of the galaxy's light that reaches a given area of Earth in a given time. Because the area and the time are constant, the radiant flux depends only on the amount of light that reaches Earth: the galaxy's apparent intensity. The inverse-square law of light says that the apparent intensity of a light source decreases proportionally to the square of the distance to the source.

This is because the amount of light the galaxy puts out – the galaxy's true intensity – is constant, but the light spreads out as it leaves the galaxy. It spreads into the surface of a sphere with surface area

$$A = 4pR^2,$$

where R is the radius of the sphere. To find the apparent intensity from Earth, think about a sphere whose center is at the galaxy and whose surface intersects the Earth. This sphere's radius is equal to the distance from the galaxy to Earth. So the apparent intensity I is proportional to the distance to Earth squared, and the radiant flux F is proportional to I:

$$F = k_1 I = k_2(4pd^2).$$

Since you only have relative distances, you have no information on the values of the constants k_1 and $4\pi k_2$, and can only find relative fluxes. If k_1 and $4\pi k_2$ can be anything, they might as well be 1. If the constants are set to 1, the equation becomes:

$$F = d^2,$$

and solving for d gives

$$d = \frac{1}{\sqrt{F}}.$$

Lastly, substituting for radiant flux from the definition of magnitude gives

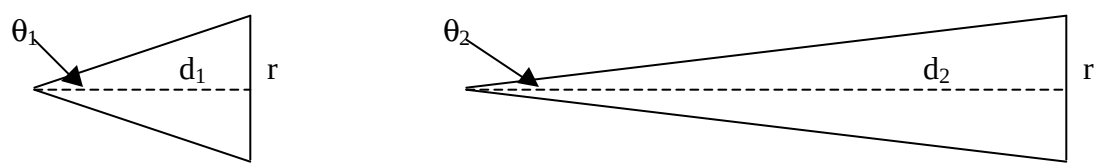
$$d = \frac{1}{\sqrt{2.51^{-m}}}.$$

From apparent size,

$$d = \frac{1}{s_{app}},$$

where s is the distance across the galaxy's image.

The galaxy has some true size; suppose that the distance across the galaxy's true size is r. The galaxy's apparent size r_{app}, depends on its angular size θ when viewed from Earth. The sketch below shows the relationship between r, θ, and d, distance from Earth for galaxies at two distances, d_1 and d_2:



The diagrams show that as d increases, θ gets smaller.

By the laws of trigonometry,

$$\tan q = \frac{r}{d}.$$

But galaxies are so far away from Earth that r is always much, much less than d. When $\tan q << 1$, you can use the approximation $\tan q \approx q$. Using this approximation, the equation becomes

$$q = \frac{r}{d}.$$

Solving for d gives

$$d = \frac{r}{q}.$$

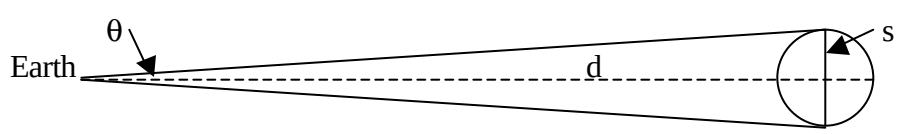
The galaxy's apparent size, r_{app}, is proportional to the angle θ with a constant of proportionality that depends on the galaxy's true size. Since you don't know the true size, and since you are only looking for relative distances anyway, you can let the constant equal 1. For the same reason, you can let the galaxy's true size r (which you don't know and don't need to know to find relative distance) equal 1. Therefore,

$$d = \frac{1}{s_{app}}.$$

Question 2: If the distance to a galaxy found from magnitudes do not agree with its distance found with apparent sizes, then the galaxy is unusual in some way. Either it is brighter/fainter than a typical galaxy of its size, or it is larger/smaller than a typical galaxy of its brightness.

Question 3: Although clusters are large objects, the universe is a very big place. Just as stars in one galaxy are much closer to each other than the galaxy is to another galaxy, galaxies in one cluster are much closer to each other than the cluster is to another cluster. The distance between two stars in a galaxy is always much, much less than the distance between two galaxies; similarly, the distance between two clusters is always much, much less than the distance between two clusters. Therefore, one can say that a single cluster is at a definite distance from Earth, and each galaxy in that cluster is at approximately that distance.

Exercise 9. The fractional error in the assumption that galaxies are at the same distance is equal to the longest distance between two galaxies in the cluster divided by the assumed distance to the cluster from Earth. Clusters are approximately spherical; suppose a cluster has diameter s and is located at distance d from Earth. The sketch below shows such a cluster:



The angular size of the cluster measured from Earth is represented by θ. All the galaxies in the cluster are within the circle of diameter s. Therefore, the galaxy nearest to Earth is at distance d - s, and the galaxy farthest from Earth is at distance d + s. Subtracting these, the error in distance is s, so the fractional error is $\frac{d}{s}$.

The laws of trigonometry say that

$$\tan q = \frac{d}{s}.$$

But since galaxy clusters are so far from Earth, d is much, much, much greater than s, so $\frac{d}{s}$ <<<< 1. Therefore, one can use the small angle approximation tan $q \approx q$. Using this approximation, $q = \frac{d}{s}$. Therefore, the fractional error in the assumption that the entire cluster is at distance d is equal to q, the angular size of the cluster measured from Earth.

Question 4. There are many possible answers to this question. Large cities have more buildings in the same area than small towns do. Usually, they also have taller buildings than small towns. They may have certain types of buildings, such as hospitals or airports, that small towns lack.

Similarly, large clusters have many more galaxies in the same area than small groups do. Unlike in the towns example, there is no correlation between galaxy size and cluster size – larger clusters do not necessarily have larger galaxies. However, large clusters are more likely to have rare galaxies, like the "rare buildings" – hospitals and airports – in large cities. This is because clusters have larger numbers of galaxies, and by looking at larger numbers of galaxies, you are more likely to find a rare galaxy.

Exercise 10. Most galaxies within the same cluster have similar colors, sizes, and brightnesses. Clusters can have different types of galaxies, although more distant clusters tend to have more elliptical galaxies.

Question 5. If Hubble and Humason saw that one approach gave them a smaller data scatter – that is, the redshift-distance data they found came closer to graphing a straight line – they would know that that particular approach was a better way to measure relative distance.

Exercise 11. The largest, whitest galaxies belong to the nearest cluster. The smaller, more bluish galaxies belong to the medium cluster. The small, faint, reddish galaxies belong to the most distant cluster.

Exercise 12. Your answers will vary depending on which galaxies you selected and what measures of relative distance you chose. Because you will need galaxies with redshifts available, you should have selected at least the ten galaxies in the table below.

The table lists magnitudes in the green wavelength and the relative distances calculated from those magnitudes. Using green-wavelength magnitudes of these ten galaxies is a

simple way to find relative distance, but it produces large data scatter. If you use a different method, you will probably find better estimates of relative distance.

Galaxy ID	Right Ascension	Declination	Green Magnitude	Relative Distance
2255031254843512	178.152981	0.915194	16.36	1
2255031254843527	178.181586	0.974817	17.42	1.63
2255048446836831	178.159608	1.06589	17.92	2.04
2255048446902406	178.284604	1.122476	18.09	2.21
2255031254843544	178.223124	0.894348	18.34	2.48
2255031254974636	178.379048	0.972349	20.04	5.44
2255031254909042	178.260686	1.040557	20.08	5.53
2255031254909306	178.298469	1.013883	20.43	6.51
2255031254909183	178.270379	1.0284	20.75	7.54
2255031254909137	178.250333	1.019779	20.79	7.68

Exercise 13. You should use a different method to find relative distances. Your distances should be roughly similar to the distances you found in exercise 12.

Exercise 14: You should find the entries in the table with the spectrum. The correct spectral lines are marked H_α, H_β, H_γ, and H_δ.

Question 6. Using the Doppler shift interpretation, a redshift of z > 1 would correspond to a velocity greater than the speed of light. Traveling faster than the speed of light is impossible because of Einstein's Special Theory of Relativity. But using the cosmological interpretation, a redshift greater than one simply means that $\frac{d(0)}{d(z)} > 2$; in other words, at the time corresponding to redshift z, the separation between two galaxies was less than half of its current separation. There is no conceptual problem with galaxies being twice as close to one another in the past. Therefore, there is no conceptual problem with z > 1.

Exercise 15. You should find the following redshifts:

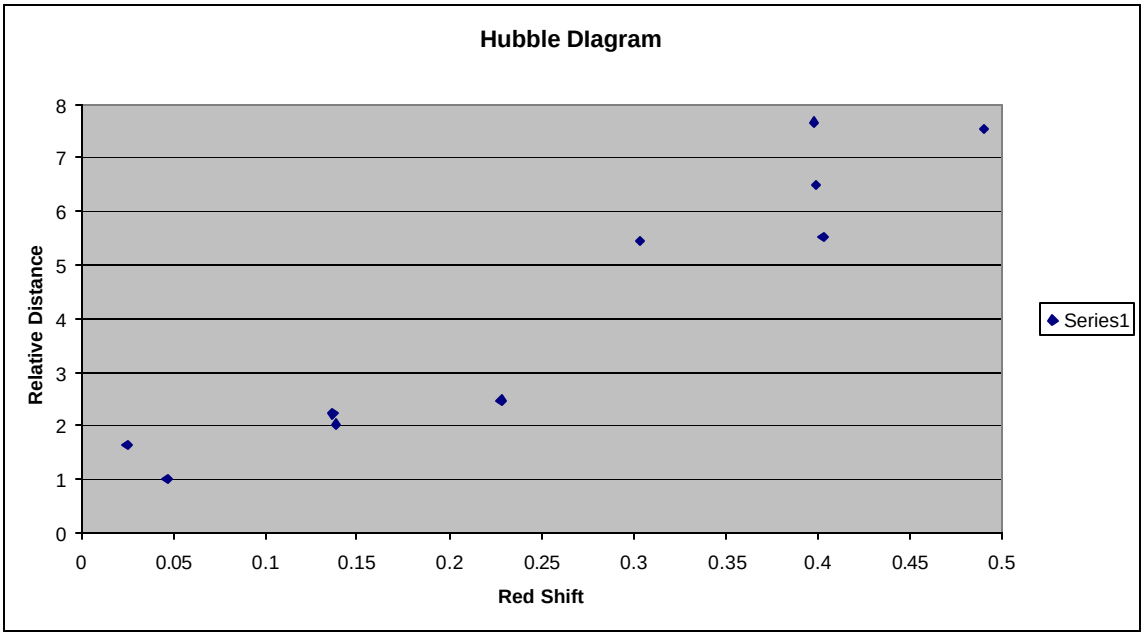
Spectrum Number	Redshift
447	0.137
453	0.024
455	0.047
522	0.137
523	0.399
525	0.398
526	0.227
527	0.49
530	0.403
580	0.303

Exercise 16. Results from Exercise 15 should be similar to the values given by Get Spectra.

Exercise 17. The average redshift for the galaxies in Exercise 15 is 0.257. The average redshift of all the galaxies in the SDSS database is about 0.1. Galaxies range from a redshift of about 0.01 to about 0.5. Quasars have an average redshift of about 1.5. Their redshifts range from about 0.1 to about 6.

Exercise 18. The inverse of H_0 = 70 km/sec/Mpc suggests that the universe is 14 billion years old. With the 10% uncertainty, the universe could be between 12.6 billion and 15.4 billion years old. This is generally consistent with stellar ages of 11 to 13 billion years, although there is some small overlap.

Exercise 19. Using the values of relative distance obtained from the green wavelength magnitudes gives the following Hubble diagram:

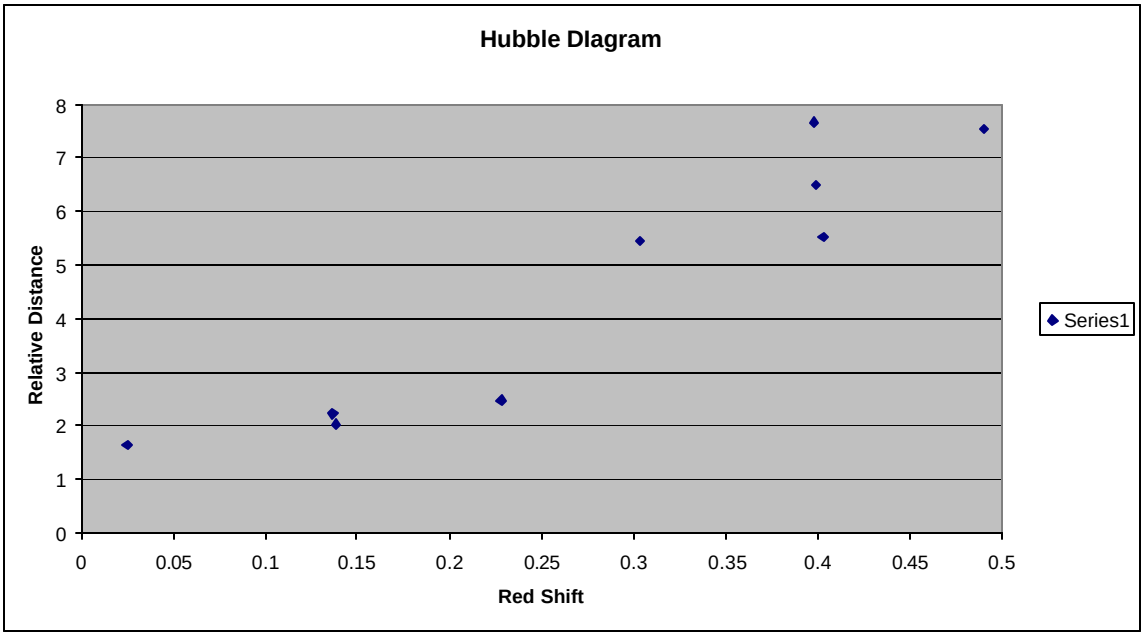


A trendline through this data has a fit of 90.8%, meaning a straight line is a good fit to the data.

If you used a different method to find the relative distance, your diagram would look slightly different.

Exercise 20. Galaxy ID 2255031254843527 is unusually dim for a galaxy at its distance. Because it is dim, using its relative magnitude to measure distance makes the galaxy appear farther from Earth than it actually is.

Here is the Hubble diagram with galaxy ID 2255031254843527 removed:



The fit has improved slightly, to 91.8%.

Question 7. The Hubble diagram indicates that all galaxies are moving away from us, and that the farther a galaxy is, the faster it moves. You should realize that to get from this observation to the expanding universe picture, you need to assume that the universe is fairly uniform on a large scale – that is, that galaxies can be found in roughly equal amounts in all directions. You also need to establish that galaxies are redshifted in all directions of the sky. If Earth were moving through the universe, then behind us, we would see redshifts, and ahead of us, we would see blueshifts. If the universe as a whole were expanding, we would see redshifts wherever we looked.

You could test both these assumptions with Skyserver. You could make a Hubble diagram in many different parts of the sky to verify that redshifts appear everywhere. You could also count galaxies in different parts of the sky to verify that galaxies are roughly uniformly distributed.